The electromagnetic forces or torques developed in the driving motor tend to propagate motion of the drive system. This motion may be uniform if the linear velocity (in the case of translational motion) or the angular velocity (in the case of rotational motion) is constant, or non-uniform, as it occurs while starting, braking or changing the load on the drive.

In case of uniform motion the torque developed by the driving motor is to overcome any resisting torque offered by the driven equipment as well as the torque due to friction. In other words, only static resisting torques, commonly called as load torques, are to be counterbalanced, if the motion were uniform.

2.1 TYPES OF LOADS

Loads can be of two types—those which provide active torques and those which provide passive torques.

Active torques are due to either gravitational force or deformation in elastic bodies. The active torques due to gravitational pull are obtained in case of hoists, lifts or elevators and railway locomotives operating on gradients. Such torques are also developed during compression or release of springs. Since the functioning of hoisting mechanisms, operation of locomotives on gradients and compression or release of springs are all associated with a change in potential energy of the drive, active torques are also closely connected to the potential energy. When a load is moved upwards or a spring is compressed, the stored potential energy increases and the active torque developed opposes the action that takes place, i.e., the torque is directed against the upward movement or compression. On the other hand, when a load is brought downwards or a spring is released the stored potential energy decreases and torque associated with it aids the action. Thus, it can be seen that the active torques continue to act in the same direction even after the direction of the drive has been reversed.

Passive torques are those due to friction or due to shear and deformation in inelastic bodies (lathes, fans, pumps etc.). They always oppose motion, retarding the rotation of the driven machine. Moreover, with change in direction of motion, the sense of torque also changes. For example, when a weight is being lifted up, the friction torque adds to the useful torque, but when lowered down it subtracts from the latter.
Chapter 2: Quadrantal Diagram of Speed-Torque Characteristics

In view of the fact that both active and passive load torques can be present in general, in a drive system, the motor driving the load may operate in different regimes—not only as a motor, but for specific periods, also as a generator and as a brake. Further, in many applications, the motor may be required to run in both directions. Therefore, in sketching the speed-torque characteristics of either the load or the motor, it is preferable to use all four quadrants of the speed-torque plane for plotting, rather than to confine the characteristics to the first quadrant alone. When drawn in this manner, the diagram is referred to as a quadrantal diagram.

The conventions used for positive and negative values of speed, motor torque and load torque in a diagram of this type must be understood very clearly. The speed is assumed to have a positive sign, if the direction of rotation is anticlockwise or in a such a way to cause an ‘upward’ or ‘forward’ motion of the drive. In the case of reversible drives, the positive sign for speed may have to be assigned arbitrarily either to anticlockwise or clockwise direction of rotation.

The motor torque is said to be positive if it produces an increase in speed in the positive sense. The load torque is assigned a positive sign when it is directed against the motor torque.

Figure 2.1 shows the four quadrant operation of a motor driving a hoist consisting of a cage with or without load, a rope wound onto a drum to hoist the cage and a balance weight of magnitude greater than that of the empty cage but less than that of the loaded one. The arrows in this figure indicate the actual directions of motor torque, load torque and motion in the four quadrants. It can be easily seen that they correspond to the sign conventions stated earlier for speed, motor torque and load torque.

The load torque of the hoisting mechanism may be assumed to be constant, i.e., independent of speed, since the forces due to friction and windage are small in the case of low speed hoists and the torque is primarily due to the gravitational pull on the cage. This torque being an active load torque doesn’t change its sign even when the direction of rotation of the driving motor is reversed. Therefore, the speed-torque curves of a hoist load can be represented by means of vertical lines passing through two quadrants. The speed-torque characteristic of a loaded hoist is shown in Fig. 2.1 by means of the vertical line passing through the first and fourth quadrants. Since the counterweight is assumed to be heavier than the empty cage, the inherent tendency of the load, viz., the empty cage is to move in an opposite direction to that of load presented by the loaded cage and hence the speed-torque curve of the unloaded hoist is represented by the vertical line passing through the second and third quadrants.

In the first quadrant, the load torque acts in a direction opposite to that of rotation. Hence, to drive the loaded hoist up, the developed torque in the motor \( T_M \) must act in the same direction as the speed of rotation, i.e., \( T_M \) should be of positive sign. Since the speed is also of positive sign, being an upward...
motion, the power will also have a positive sign, i.e., the drive is said to be ‘motoring’. Quadrant I is arbitrarily and conventionally, thus, designated as ‘forward motoring quadrant’.

Fig. 2.1. Four quadrant operation of a motor driving a hoist load

The hoisting up of the unloaded cage is represented in the second quadrant. Since the counterweight is heavier than the empty cage, the speed at which the hoist is moved upwards may reach a dangerously high value. In order to avoid this, the motor torque must act opposite to the direction of rotation, i.e., the motor should switch over to a braking or generator regime. Note that $T_M$ will have a negative sign and speed still has a positive sign, giving power a negative sign, corresponding to the generator or braking operation.

The third quadrant represents the downward motion of the empty cage. The downward journey of the cage is prevented by the torque due to the counterweight and friction at the transmitting parts. Therefore, in order to move the cage downwards, the motor torque must act in the same direction as the motion of the cage. The electrical machine acts as a motor as in the first quadrant, but in the reverse direction. Thus, quadrant III becomes ‘reverse motoring’. The motor torque has a negative sign (because it causes an increase in speed in the negative sense) and the speed also has a negative sign (being a downward motion). Power, thus, has a positive sign.
The downward motion of the loaded cage is shown in the fourth quadrant. The motion can take place under the action of load itself, without the use of any motor. But, the speed of downward motion can be dangerously high. Therefore, in this case, the electrical machine must act as a brake limiting the speed of the downward motion of the hoist. The motor torque has a positive sign since it causes a decrease in speed in the downward motion. The speed, of course, has a negative sign, being a downward journey. The power, thus, acquires a negative sign, corresponding to the braking operation of the motor.

A second basic type of loading that occurs is the one characterized by dry friction. This type of load presents to the motor a passive torque, which is essentially independent of speed. It is characterized also by the requirement of an extra torque at very near zero speed. In power applications it is, often, called as the break away torque and in control systems, it is referred to as stiction (derived from sticking friction). The speed-torque curves for this type of load are shown in Fig. 2.2.

Another type of friction loading is used by control system engineers and is known as viscous friction. It is a force or torque loading whose magnitude is directly proportional to the speed. The viscous friction torque speed curves are illustrated in Fig. 2.3. Calendering machines, Eddy current brakes and separately excited dc generators feeding fixed resistance loads have such speed-torque characteristics.

Yet another basic type of load torque is one whose magnitude is proportional to some power of the speed. Such a load is best illustrated by a fan or blower. The torque produced by the fan is directly proportional to the square of the speed throughout the range of usable fan speeds. The speed-torque curves for the fan type of load are presented in Fig. 2.4. Centrifugal pumps, propellers in ships or aeroplanes also have the same type of speed-torque characteristic.

Hyperbolic speed-torque characteristic (load torque being inversely proportional to speed or load power remaining constant), as shown in Fig. 2.5, is associated with certain type of lathes, boring machines, milling machines, steel mill coilers, etc.
In general, the load torque in any specified application may consist of any of the above mentioned loads in varying proportions.

### 2.3 Load Torques That Depend on the Path or Position Taken by the Load During Motion

In the preceding section, we have been considering load torques which vary as a function of speed. However, load torques, that depend not only on speed but also on the nature of the path traced out by the load during its motion, are present both in hoisting mechanisms and transport systems. For instance, the resistance to motion of a train travelling upgradient or taking a turn depends on the magnitude of the gradient or the radius of curvature of the track respectively.

The force resisting the motion of a train travelling upgradient, as shown in Fig. 2.6 is given by

\[
F_G = W \sin \alpha \approx W \tan \alpha \quad (\alpha, \text{being usually small})
\]

\[
= W \frac{G}{1000} \text{kg,} \quad \ldots(2.1)
\]

where \(W\) = dead weight of the train or any other transport system, in kg, and \(G\) = gradient expressed as a rise in meters in a track distance of 1000 metres.
The tractive force required to overcome curve resistance is given by the empirical formula stated below:

\[ F_c = \frac{700}{R} W \text{ kg} \]  

...(2.2)

where \( R \) is the radius of curvature in metres.

In hoisting mechanisms in which tail ropes or balancing ropes are not used (Fig. 2.7) the load torque is not only due to the weight of the unloaded or the loaded cage but also due to that of the lifting ropes or cables. The latter depends on the position of the two cages. When cage 1 is at the bottom most position and is to be lifted upwards, the entire weight of the rope is also to be moved up. When both cages remain at the same height, the weight of the rope to be lifted up becomes zero, since the weight of the ropes on both sides balance each other. When cage 1 is at a higher position than cage 2, a portion of the weight of the rope acts in such a way as to aid the upward motion of cage 1. In fact, when cage 1 occupies the topmost position, the whole weight of the rope aids the upward movement.

The force that resists the upward motion of the load \( F_r \) due to the varying weight of the rope depending on the position of the load, is given as:

\[ F_r = W_r \left(1 - \frac{2x}{h}\right) \text{ kg} \]  

...(2.3)

where \( W_r \) = total weight of the rope, in kg, 
\( x \) = height of the cage at any arbitrary position from the bottom most position in m, and 
\( h \) = the desired maximum height to which the cage is to be moved upwards, in m.

Since, for very high values of ‘\( h \)’, the weight of the rope may be considerably greater than that of the load to be lifted upwards, the force \( F_r \) affects, to a large extent, the performance of the drive used in hoisting mechanisms. By using tail ropes, as shown by means of dotted lines in Fig. 2.7, the weight of the connecting rope can be balanced and more or less smooth movement of the cages can be ensured.

Another example of a load torque, which depends on path traced out during motion, is that of a planing machine. At a particular position of the moving table containing the workpiece, the load torque comes in the form of a sudden blow; in a different position, after the cutter has come out of the job, the magnitude of the load torque decreases sharply.

### 2.4 LOAD TORQUES THAT VARY WITH ANGLE OF DISPLACEMENT OF THE SHAFT

In all machines, having crankshafts, for example, in reciprocating pumps and compressors, frame saws, weaving looms, rocking pumps used in petroleum industry etc., load torque is a function of the position of the crank, i.e., the angular displacement of the shaft or rotor of the motor. Load torque in drives used for steering ships also belongs to this category.
Figure 2.8 shows the approximate relationship between the load torque and angular displacement of the shaft \( \theta \) for a reciprocating compressor. It is of the form \( T_L = f(\theta) \), where \( \theta \) itself varies with time. For all such machines, the load torque \( T_L \) can be resolved into two components—one of constant magnitude \( T_{av} \) and the other a variable \( T'_L \), which changes periodically in magnitude depending on the angular position of the shaft. Such load torque characteristics, can, for simplicity, be represented by

\[
T' = \sum_{r=0}^{m} T_{lr} \sin(r\theta + \phi_r) \quad \text{...(2.4)}
\]

\( \theta = \omega t \), where \( \omega \) represents the angular speed of the shaft of the motor driving the compressor.

During changes in speed, since only small deviations from a fixed value of speed \( \omega_a \) occur, the angular displacement can be represented by \( \theta = (\omega_a + \Delta \omega)t \). Therefore, the variable portion of the load torque may be expressed as

\[
T' = \sum_{r=0}^{m} T_{lr} \sin[(r\omega_a t + \phi_r) + r\Delta \omega t] \quad \text{...(2.5)}
\]

The term \( r\Delta \omega t \) being of very small magnitude can be neglected. Thus, restricting to small deviations in angle from the equilibrium position, a load torque which varies with the angular displacement of the shaft can be transformed to one which varies periodically with respect to time.

### 2.5 LOAD TORQUES THAT VARY WITH TIME

Of equal or perhaps greater importance in motor selection is the variation of load torque with time. This variation, in certain applications, can be periodic and repetitive, one cycle of variation being called a duty cycle. It is convenient to classify different types of loads under the following groups:

- **(a) Continuous, constant loads**: Centrifugal pumps or fans operating for a long time under the same conditions; paper-making machines etc.
- **(b) Continuous, variable loads**: Metal cutting lathes; hoisting winches; conveyors etc.
- **(c) Pulsating loads**: Reciprocating pumps and compressors; frame saws, textile looms and, generally, all machines having crank shaft.
(d) *Impact loads*: Apparent, regular and repetitive load peaks or pulses which occur in rolling mills, presses, shearing machines, forging hammers etc. Drives for such machines are characterized by heavy flywheels.

(e) *Short time intermittent loads*: Almost all forms of cranes and hoisting mechanisms; excavators; roll trains etc.

(f) *Short time loads*: Motor-generator sets for charging batteries; servomotors used for remote control of clamping rods of drilling machines.

Certain machines like stone crushers and ball mills do not strictly fall under any of the above groups. If these loads were characterized by frequent impacts of comparatively small peaks, it would be more appropriate to classify them under continuous variable loads rather than under impact loads. Sometimes, it is difficult to distinguish pulsating loads from impact loads, since both of them are periodic in nature and, hence, may be expressed as a sum of sinusoidal waves of different amplitude, frequency and phase.

One and the same machine can be represented by a load torque which varies either with speed or with time. For example, a fan load whose load torque is proportional to the square of the speed, is also a continuous, constant load.
Load torque of a crane is independent of speed and also short time intermittent in nature. Rocking pumps for petroleum have a load which vary with angular position of the shaft, but can also be classified as a pulsating load.

The nature of load (power) variation with respect to time corresponding to certain common applications is shown in Fig. 2.9.

### 2.6 DYNAMICS OF MOTOR-LOAD COMBINATION

The motor and the load that it drives can be represented by the rotational system shown in Fig. 2.10. Although the load, in general, may not rotate at the same speed as the motor, it is convenient to represent it in this manner so that all parts of the motor-load system have the same angular velocity. In case, the speed of the load differs from that of the motor, one can find out an equivalent system (as explained later).

The basic torque equation, known as the equation of motion, for the above motor-load system, is written as

\[ T_M = T_L + J \frac{d\omega}{dt} \]  

where \( T_M \) and \( T_L \) denote motor and load torque measured in N-m; \( J \), the moment of inertia of drive system in kg-m\(^2\) and \( \omega \), the angular velocity in mechanical radians/sec.

In the above equation the motor torque is considered as an applied torque and the load torque as a resisting torque.

From the above equation, it is possible to determine the different states at which an electric drive causing rotational motion can remain.
(i) $T_M > T_L$, i.e., $d\omega/dt > 0$, i.e., the drive will be accelerating, in particular, picking up speed to reach rated speed.

(ii) $T_M < T_L$, i.e., $d\omega/dt < 0$, i.e., the drive will be decelerating and, particularly coming to rest.

(iii) $T_M < T_L$, i.e., $d\omega/dt = 0$, i.e., the motor will continue to run at the same speed, if it were running or will continue to be at rest, if it were not running.

The above statements, namely, that when $T_M > T_L$ the drive accelerates and that when $T_M = T_L$ the drive decelerates, are valid only when $T_L$ happens to be a passive load. The reverse may occur with active loads. For example, if we were to switch on the motor for hoisting up a winch, while it is coming down on its own weight, until the direction of rotation changes, deceleration of the drive and not acceleration takes place, when $T_M > T_L$. In case $T_M < T_L$ in the above situation when the motor has been switched on for moving the winch up, the load will continue to come down and the motor will accelerate instead of decelerating.

The term $J d\omega/dt$ which represents the inertia torque, is also known as dynamic torque, since it is present only during transient conditions, i.e., when the speed of the drive varies. During acceleration of the drive, the inertia torque is directed against motion, but during braking it maintains the motion of the drive. Thus, inertia torque is determined both in magnitude and sign, as the algebraic sum of the motor and load torques.

In view of the above, the signs for $T_M$ and $T_L$ in Eqn. (2.6) correspond to motoring operation of the driving machine and to passive load torque or to a braking torque caused by active loads, respectively. The equation of motion can, in general, be written as:

$$\pm T_M = \pm T_L + \frac{J d\omega}{dt} \quad \ldots(2.7)$$

The signs to be associated with $T_M$ and $T_L$ in Eqn. (2.7) depend, as indicated earlier, on the regime of operation of the driving motor and the nature of load torque. The equation of motion enables us to determine the variation of torque, current and speed with respect to time, during transient operation of the drive.

### 2.6.1 Equivalent System

Seldom is a motor shaft directly coupled to load shafts. In general, the different loads connected to the motor will have different speed requirements. Speed changing mechanisms such as gears, V-belts, etc., will be used to obtain different speeds. Since the ultimate objective is to select a motor suitable for the application, it is desirable to refer all mechanical quantities such as load torque, inertia torque, etc., to one single axis of rotation, conveniently, the output shaft of the motor. The principle of conservation of energy will be used for this purpose.