CHAPTER 1

Flat Slabs

1.1 INTRODUCTION

Common practice of design and construction is to support the slabs by beams and support the beams by columns. This may be called as beam-slab construction. The beams reduce the available net clear ceiling height. Hence in warehouses, offices and public halls some times beams are avoided and slabs are directly supported by columns. This types of construction is aesthetically appealing also. These slabs which are directly supported by columns are called **Flat Slabs.** Fig. 1.1 shows a typical flat slab.

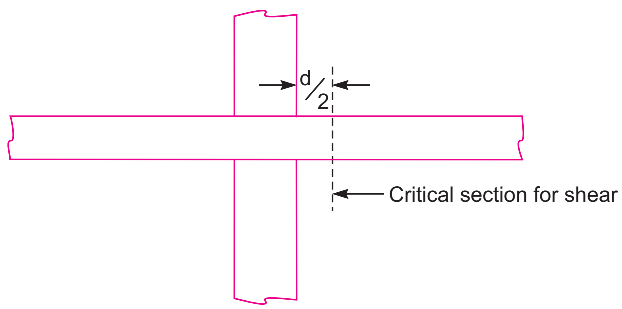


Fig. 1.1 *A typical flat slab (without drop and column head)*

The column head is some times widened so as to reduce the punching shear in the slab. The widened portions are called **column heads.** The column heads may be provided with any angle from the consideration of architecture but for the design, concrete in the portion at 45º on either side of vertical only is considered as effective for the design [Ref. Fig. 1.2].

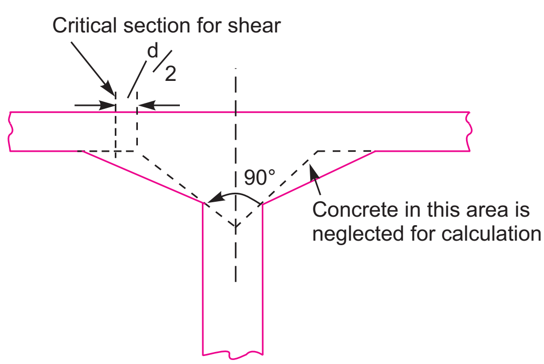


Fig. 1.2 *Slab without drop and column with column head*

Moments in the slabs are more near the column. Hence the slab is thickened near the columns by providing the drops as shown in Fig. 1.3. Sometimes the drops are called as capital of the column. Thus we have the following types of flat slabs:

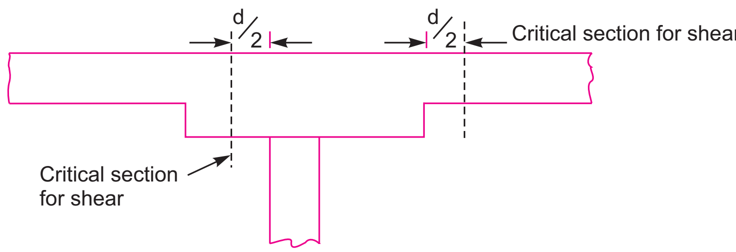


Fig. 1.3 *Slab with drop and column without column head*

(*i*) Slabs without drop and column head (Fig. 1.1).
(*ii*) Slabs without drop and column with column head (Fig. 1.2).
(*iii*) Slabs with drop and column without column head (Fig. 1.3).
(*iv*) Slabs with drop and column head as shown in Fig. 1.4.

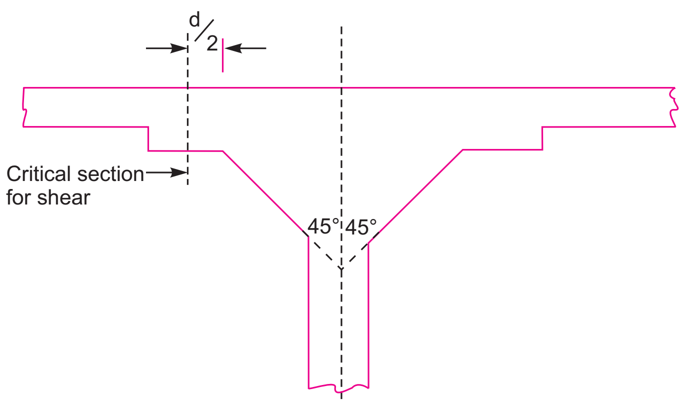


Fig. 1.4 *Slab with drop and column with column head*

The portion of flat slab that is bound on each of its four sides by centre lines of adjacent columns is called a panel. The panel shown in Fig. 1.5 has size $L_1 \times L_2$. A panel may be divided into column strips and middle strips. Column Strip means a design strip having a width of $0.25L_1$ or $0.25L_2$, whichever is less. The remaining middle portion which is bound by the column strips is called middle strip. Fig. 1.5 shows the division of flat slab panel into column and middle strips in the direction *y*.

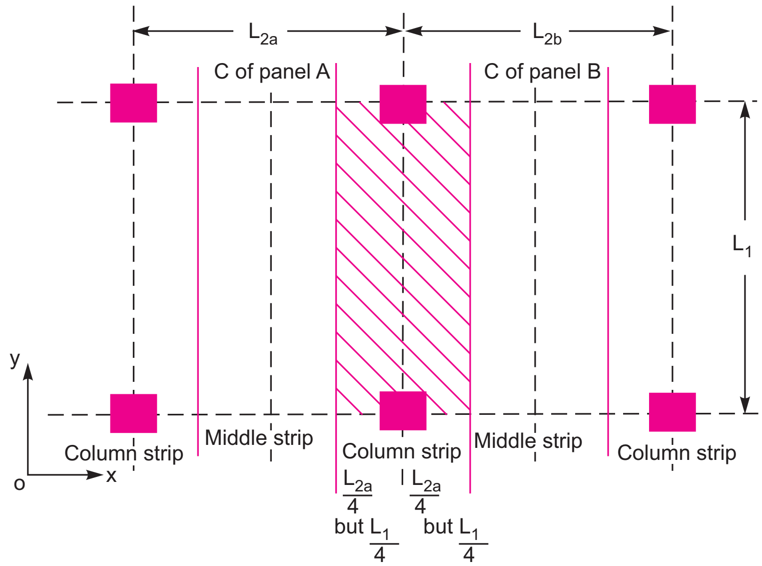


Fig. 1.5 *Panels, column strips and middle strips in y-direction*

1.2 PROPORTIONING OF FLAT SLABS

IS 456-2000 [Clause 31.2] gives the following guidelines for proportioning.

1.2.1 Drops

The drops when provided shall be rectangular in plan, and have a length in each direction not less than one third of the panel in that direction. For exterior panels, the width of drops at right angles to the non continuous edge and measured from the centre-line of the columns shall be equal to one half of the width of drop for interior panels.

1.2.2 Column Heads

Where column heads are provided, that portion of the column head which lies within the largest right circular cone or pyramid entirely within the outlines of the column and the column head, shall be considered for design purpose as shown in Figs. 1.2 and 1.4.

1.2.3 Thickness of Flat Slab

From the consideration of deflection control IS 456-2000 specifies minimum thickness in terms of span to effective depth ratio. For this purpose larger span is to be considered. If drop as specified in 1.2.1 is provided, then the maximum value of ratio of larger span to thickness shall be

= 40, if mild steel is used

= 32, if Fe 415 or Fe 500 steel is used

If drops are not provided or size of drops do not satisfy the specification 1.2.1, then the ratio shall not exceed 0.9 times the value specified above *i.e.*,

$= 40 \times 0.9 = 36$, if mild steel is used.

$= 32 \times 0.9 = 28.8$, if HYSD bars are used

It is also specified that in no case, the thickness of flat slab shall be less than 125 mm.

1.3 DETERMINATION OF BENDING MOMENT AND SHEAR FORCE

For this IS 456-2000 permits use of any one of the following two methods:

(*a*) The Direct Design Method

(*b*) The Equivalent Frame Method

1.4 THE DIRECT DESIGN METHOD

This method has the limitation that it can be used only if the following conditions are fulfilled:

(*a*) There shall be minimum of three continuous spans in each directions.

(*b*) The panels shall be rectangular and the ratio of the longer span to the shorter span within a panel shall not be greater than 2.

(*c*) The successive span length in each direction shall not differ by more than one-third of longer span.

(*d*) The design live load shall not exceed three times the design dead load.

(*e*) The end span must be shorter but not greater than the interior span.

(*f*) It shall be permissible to offset columns a maximum of 10 percent of the span in the direction of the offset not withstanding the provision in (*b*).

Total Design Moment

The absolute sum of the positive and negative moment in each direction is given by

$$M_0 = \frac{WL_n}{8}$$

Where,

M_0 = Total moment

W = Design load on the area $L_2 \times L_n$

L_n = Clear span extending from face to face of columns, capitals, brackets or walls but not less than 0.65 L_1

L_1 = Length of span in the direction of M_0; and

L_2 = Length of span transverse to L_1

In taking the values of L_n, L_1 and L_2, the following clauses are to be carefully noted:

(*a*) Circular supports shall be treated as square supports having the same area *i.e.*, squares of size 0.886D.

(*b*) When the transverse span of the panel on either side of the centre line of support varies, L_2 shall be taken as the average of the transverse spans. In Fig. 1.5 it is given by $\frac{(L_{2a} + L_{2b})}{2}$.

(*c*) When the span adjacent and parallel to an edge is being considered, the distance from the edge to the centre-line of the panel shall be substituted for L_2.

Distribution of Bending Moment into –ve and +ve Moments

The total design moment M_0 in a panel is to be distributed into –ve moment and +ve moment as specified below:

In an Interior Span

Negative Design Moment	$0.65\ M_0$
Positive Design Moment	$0.35\ M_0$

In an End Span

Interior negative design moment

$$= \left[0.75 - \frac{0.10}{1 + \frac{1}{\alpha_c}}\right] M_0$$

Positive design moment

$$= \left[0.63 - \frac{0.28}{1 + \frac{1}{\alpha_c}}\right] M_0$$

Exterior negative design moment

$$= \left[\frac{0.65}{1 + \frac{1}{\alpha_c}}\right] M_0$$

where α_c is the ratio of flexural stiffness at the exterior columns to the flexural stiffness of the slab at a joint taken in the direction moments are being determined and is given by

$$\alpha_c = \frac{\sum K_c}{K_s}$$

Where,

$\sum K_c$ = Sum of the flexural stiffness of the columns meeting at the joint; and

K_s = Flexural stiffness of the slab, expressed as moment per unit rotation.

Distribution of Bending Moments Across the Panel Width

The +ve and –ve moments found are to be distributed across the column strip in a panel as shown in Table 1.1. The moment in the middle strip shall be the difference between panel and the column strip moments.

Table 1.1 Distribution of Moments Across the Panel Width in a Column Strip

S. No.	*Distributed Moment*	*Per cent of Total Moment*
a	Negative BM at the exterior support	100
b	Negative BM at the interior support	75
c	Positive bending moment	60

Moments in Columns

In this type of constructions column moments are to be modified as suggested in IS 456–2000 [Clause No. 31.4.5].

Shear Force

The critical section for shear shall be at a distance $\frac{d}{2}$ from the periphery of the column/capital drop panel. Hence if drops are provided there are two critical sections near columns. These critical sections are shown in Figs. 1.1 to 1.4. The shape of the critical section in plan is similar to the support immediately below the slab as shown in Fig. 1.6.

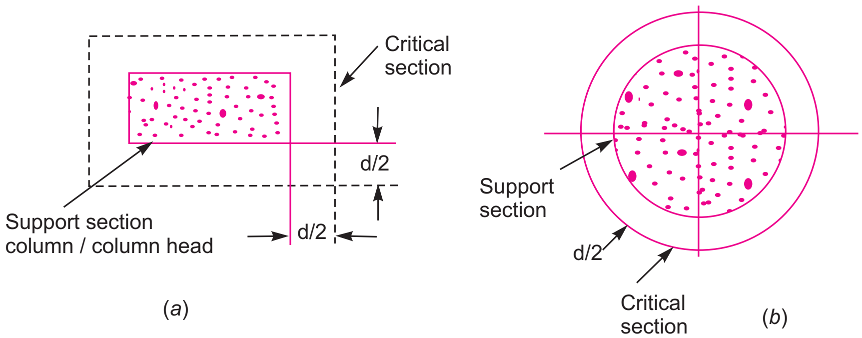


Fig. 1.6

For columns sections with re-entrant angles, the critical section shall be taken as indicated in Fig. 1.7.

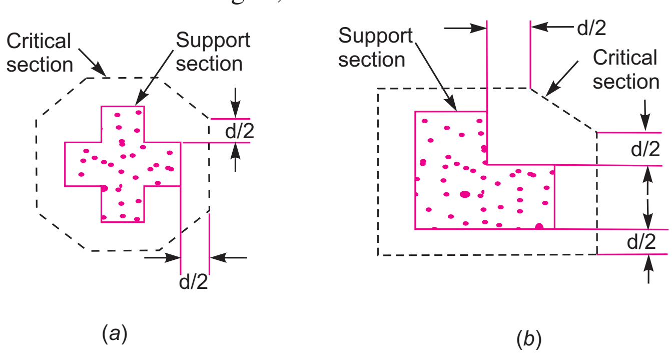


Fig. 1.7

In case of columns near the free edge of a slab, the critical section shall be taken as shown in Fig. 1.8.

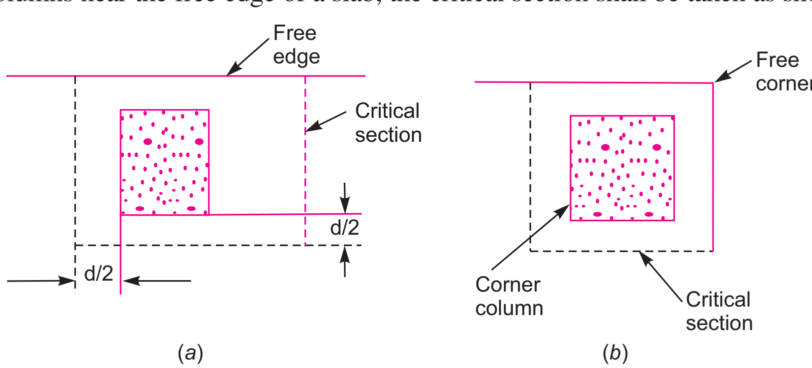


Fig. 1.8

The nominal shear stress may be calculated as

$$\tau_v = \frac{V}{b_0 d}$$

where V – is design shear force

b_0 – is the periphery of the critical section

d – is the effective depth

The permissible shear stress in concrete may be calculated as $k_s\ \tau_c$, where $k_s = 0.5 + \beta_c$ but not greater than 1, where β_c is the ratio of short side to long side of the column/capital; and

$$\tau_c = 0.25\sqrt{f_{ck}}$$

If shear stress $\tau_v < \tau_c$ – no shear reinforcement are required. If $\tau_c < \tau_v < 1.5\ \tau_c$, shear reinforcement shall be provided. If shear stress exceeds 1.5 τ_c flat slab shall be redesigned.

1.5 EQUIVALENT FRAME METHOD

IS 456–2000 recommends the analysis of flat slab and column structure as a rigid frame to get design moment and shear forces with the following assumptions:

(*a*) Beam portion of frame is taken as equivalent to the moment of inertia of flat slab bounded laterally by centre line of the panel on each side of the centre line of the column. In frames adjacent and parallel to an edge beam portion shall be equal to flat slab bounded by the edge and the centre line of the adjacent panel.

(*b*) Moment of inertia of the members of the frame may be taken as that of the gross section of the concrete alone.

(*c*) Variation of moment of inertia along the axis of the slab on account of provision of drops shall be taken into account. In the case of recessed or coffered slab which is made solid in the region of the columns, the stiffening effect may be ignored provided the solid part of the slab does not extend more than 0.15 l_{ef} into the span measured from the centre line of the columns. The stiffening effect of flared columns heads may be ignored.

(*d*) Analysis of frame may be carried out with substitute frame method or any other accepted method like moment distribution or matrix method.

Loading Pattern

When the live load does not exceed ¾th of dead load, the maximum moments may be assumed to occur at all sections when full design live load is on the entire slab.

If live load exceeds ¾th dead load analysis is to be carried out for the following pattern of loading also:

(*i*) To get maximum moment near mid span:
¾th of live load on the panel and full live load on alternate panel

(*ii*) To get maximum moment in the slab near the support:
¾th of live load is on the adjacent panel only

It is to be carefully noted that in no case design moment shall be taken to be less than those occurring with full design live load on all panels.

The moments determined in the beam of frame (flat slab) may be reduced in such proportion that the numerical sum of positive and average negative moments is not less than the value of total design moment $M_0 = \frac{WL_n}{8}$. The distribution of slab moments into column strips and middle strips is to be made in the same manner as specified in direct design method.

1.6 SLAB REINFORCEMENT

Spacing

The spacing of bars in a flat slab, shall not exceed 2 times the slab thickness.

Area of Reinforcement

When the drop panels are used, the thickness of drop panel for determining area of reinforcement shall be the lesser of the following:

(*a*) Thickness of drop, and

(*b*) Thickness of slab plus one quarter the distance between edge of drop and edge of capital.

The minimum percentage of the reinforcement is same as that in solid slab *i.e.*, 0.12 percent if HYSD bars used and 0.15 percent, if mild steel is used.

Minimum Length of Reinforcement

At least 50 percent of bottom bars should be from support to support. The rest may be bent up. The minimum length of different reinforcement in flat slabs should be as shown in Fig. 1.9 (Fig. 16 in IS 456–2000). If adjacent spans are not equal, the extension of the –ve reinforcement beyond each face shall be based on the longer span. All slab reinforcement should be anchored property at discontinuous edges.

Example 1.1: *Design an interior panel of a flat slab of size 5 m × 5 m without providing drop and column head. Size of columns is 500 × 500 mm and live load on the panel is 4 kN/m². Take floor finishing load as 1 kN/m². Use M20 concrete and Fe 415 steel.*

Solution:

Thickness

Since drop is not provided and HYSD bars are used span to thickness ratio shall not exceed $\frac{1}{0.9 \times 32} = \frac{1}{28.8}$

∴ Minimum thickness required

$$= \frac{\text{Span}}{28.8} = \frac{5000}{28.8} = 173.6 \text{ mm}$$

Let $d = 175$ mm and D = 200 mm

Loads

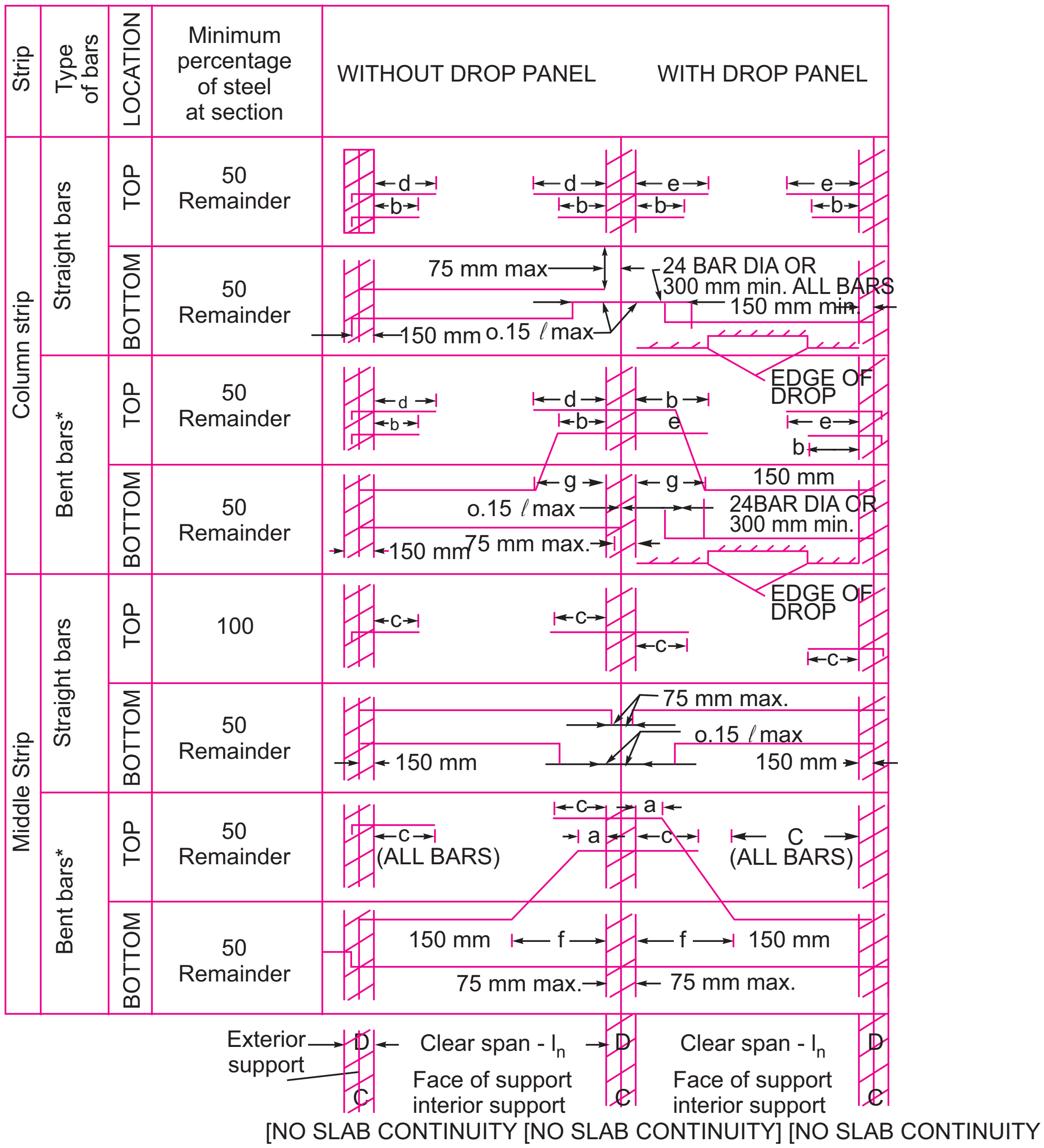
Self weight of slab $= 0.20 \times 25 = 5$ kN/m²

Finishing load $= 1$ kN/m²

Live load $= 4$ kN/m²

∴ Total working load $= 10$ kN/m²

Factored load $= 1.5 \times 10 = 15$ kN/m²



Bar Length From Face of Support							
	Minimum Length						Maximum Length
Mark	a	b	c	d	e	f	g
Length	0.14 l_n	0.20 l_n	0.22 l_n	0.30 l_n	0.33 l_n	0.20 l_n	0.24 l_n

* Bent bars at exterior supports may be used if a general analysis is made.

Note. D is the diameter of the column and the dimension of the rectangular column in the direction under consideration.

Fig. 1.9 *Minimum bend joint locations and extensions for reinforcement in flat slabs*

$$L_n = 5 - 0.5 = 4.5 \text{ m}$$

$\therefore$ Total design load in a panel $W = 15\, L_2\, L_n = 15 \times 5 \times 4.5 = 337.5$ kN

Moments

Panel Moment $\quad M_0 = \dfrac{WL_n}{8} = 337.5 \times \dfrac{4.5}{8} = 189.84$ kNm

Panel –ve moment $\quad = 0.65 \times 189.84 = 123.40$ kNm

Panel +ve moment $\quad = 0.35 \times 189.84 = 66.44$ kNm

Distribution of moment into column strips and middle strip:

	Column Strip in kNm	*Middle Strip in kNm*
–ve moment	$0.75 \times 123.40 = 92.55$	$123.40 - 92.55 = 30.85$
+ve moment	$0.60 \times 66.44 = 39.86$	$64.44 - 39.86 = 26.58$

Checking the thickness selected:

Since Fe 415 steel is used,

$$M_{u\,lim} = 0.138\, f_{ck}\, b\, d^2$$

Width of column strip $\quad = 0.5 \times 5000 = 2500$ mm

$$\therefore \quad M_{u\,lim} = 0.138 \times 20 \times 2500 \times 175^2 = 211.3125 \times 10^6 \text{ Nmm}$$

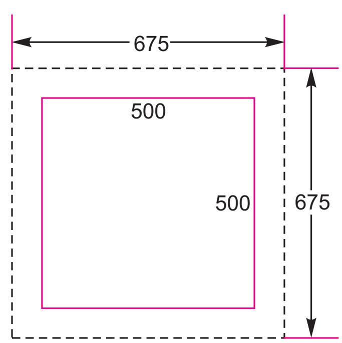
$$= 211.3125 \text{ kNm}$$

Hence singly reinforced section can be designed *i.e.*, thickness provided is satisfactory from the consideration of bending moment.

Check for Shear

The critical section for shear is at a distance $\dfrac{d}{2}$ from the column face. Hence periphery of critical section around a column is square of a size $= 500 + d = 500 + 175 = 675$ mm

Shear to be resisted by the critical section



$$V = 15 \times 5 \times 5 - 15 \times 0.675 \times 0.675$$

$$= 368.166 \text{ kN}$$

$$\therefore \quad \tau_v = \frac{368.166 \times 1000}{4 \times 675 \times 175} = 0.779 \text{ N/mm}^2$$

$k_s = 0.5 + \beta_c$ subject to maximum of 1.

$$\beta_c = \frac{L_1}{L_2} = \frac{5}{5} = 1$$

$$\therefore \quad k_s = 1$$

$$\tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.118 \text{ N/mm}^2$$

safe in shear since $\tau_v < \tau_c$