7.1 STRESS-STRAIN BEHAVIOUR OF MATERIAL

All engineering materials do not show same sort of behaviour when subjected to tension as well as compression. There exist some materials like metals, alloys etc., which are more or less equally strong in both tension and compression. And these materials are generally tested in tension again concrete, stones, bricks etc., are such type of materials which are weaker in tension and stronger in compression. Hence, these materials are tested in compression.

Now the stress-strain characteristics of mild steel are of specific importance to the community dealing with basic engineering science.

7.2 STRESS-STRAIN CHARACTERISTICS OF MILD STEEL (M.S)

In order to obtain stress-strain behaviour of M.S, a specimen of uniform circular cross-section is prepared following the specification laid in IS 1608:2005 identical to ISO 6892:1998. A specific length of maximum 4 inch or 100mm is generally selected in the well-middle part of the specimen and this length is designated as gauge length, over which the amount of elongation is studied.
Now the specimen, suitably fitted in extensometer, mounted on the machine where loading is started gradually from zero till failure. Following is a stress-strain curve of M.S specimen having gauge length 100mm, tested in Amsler Universal Testing Machine of capacity 20T. Various points on stress-strain curve are marked in Figure 7.1.

7.3 PROPORTIONAL LIMIT
It is the point on the stress-strain curve, up to which the plot is a straight line and stress is proportional to strain. Up to proportional limit, the material remains elastic and strictly follows Hooke’s Law.

7.4 ELASTIC LIMIT
In the stress-strain curve, it is the point just beyond proportional limit. From proportional limit to elastic limit, the material remains elastic but does not follow Hooke’s Law and so, stress and strain are not proportional.

7.5 YIELD POINT
When the specimen is loaded beyond elastic limit, it enters into elasto-plastic zone. In this region, elongation of specimen occurs by considerable amount without any perceivable amount of increase in load. Sometimes this yielding is accompanied by an abrupt reduction of load and thereby stress. In this case the upper and lower limits of stress are called upper yield point or stress and lower yield point or stress, respectively.

Lower yield stress is normally considered as yield stress \( \sigma_y \) of material, because upper yield stress is affected by speed of testing, form of specimen and shape of cross-section.

7.6 PROOF STRESS
Some materials like High Strength Deformed (HSD) steel, brass, duralumin etc., do not show any well defined yield point. For these materials, proof stress serves as analogous to yield stress.

Proof stress is the stress that is just sufficient to produce under load, a defined amount of permanent residual strain, which a material can have without appreciable structural damage. This arbitrary value will be different for different material or different uses of same material.

It is determined from the stress-strain curve by drawing a line parallel to initial straight part or tangent of the curve and at a distance from the origin by an amount representing the defined residual strain (normally 0.1% or 0.2%) thus determining the stress at which the line cuts the curve.
In specifying proof stress, the amount of permanent strain considered, should be mentioned, i.e., 0.1% *proof stress*, 0.2% *proof stress* etc.

### 7.7 ULTIMATE STRESS

Yield point serves as the gateway to *plastic zone*. Beyond yield point, due to sudden decrease in load, material begins to strain-harden and recover some of the elastic property. And by virtue of that, gradual uprise of stress-strain curve occurs and terminates at a point, called *ultimate stress*. This is the maximum stress, the specimen can withstand, without any appreciable damage or permanent deformation.

### 7.8 BREAKING STRESS

While ultimate stress is the maximum stress with standing capacity prior to failure, further increase of ultimate stress leads to failure of the specimen and this occurs at breaking stress. Here the value of *breaking stress* lower than ultimate stress, as appearing in the stress-strain diagram obtained during experiment, of ductile material, is somehow misleading.

What happens *in reality* is that, beyond ultimate stress, there occurs a *reduction in area* of cross-section near at the middle of gauge length. This phenomenon is called *formation of neck* or *formation of waist*. As the grips of extensometer are attached at the end of gauge length, the effect of neck formation thereby the reduction in diameter of the specimen cannot be taken into account. By reason of which breaking stress exhibits value lower than ultimate stress. And this breaking stress is called *Nominal Breaking Stress*.

When the reduced cross-sectional area at neck is considered to compute actual stress, it is found that breaking stress is pretty higher than ultimate stress. And this is called *True Breaking Stress*.

In case of brittle material, ultimate stress is same as breaking stress.

### 7.9 WORKING STRESS AND FACTOR OF SAFETY

In practical design of structures, some *uncertainties* may be associated in terms of loading, material properties etc. Not only that, in some materials, like concrete, non-ferrous alloys etc., Hooke’s Law does not hold good. To encompass all these aspects, it is essential to limit actual stress generated to a value comparatively lower than yield stress of the material. And this stress is considered as a safe one. This safe stress is designated at *Working Stress* ($\sigma_w$).

A pure number, *higher than* 1 (whole or fraction) that divides the yield stress to obtain working stress is called *Factor of Safety*.

\[ \sigma_w = \frac{\sigma_y}{n}, \text{ where } n = \text{factor of safety.} \]  \hspace{1cm} \text{...}(7.1)

Sometimes working stress is computed deviding ultimate stress by factor of safety.

### 7.10 DUCTILITY

It is the property of a material which allows of its being drawn out by tension to a small section. Britteness is the lack of ductility.

### 7.11 MALLEABILITY

It is the property of a material by virtue of which it can be turned to a very thin sheet by the application of pressure.
7.12 TOUGHNESS
It can be said, in general, resistance to deformation. This deformation may be due to impact, abrasive force, punch etc.

7.13 RESILIENCE
Let us consider a bar of length $l$ and cross-sectional area $A$ hanging vertically fixed at top, subjected to a normal pull $P$.

At zero-th instant of application of force, induced deflection is zero. Gradually load is increased within proportional limit. At position $D$, the amount of applied load is $P$ which causes a displacement $\delta$.

Here, external work done by the force, will be represented by shaded area in Figure 7.4 (b).

$$\therefore W_e = \frac{1}{2} P \cdot \delta$$  \hfill (7.2)

To resist the effect of external force, reactive work will be done by the force, generated internally due to deformation of body. This reactive work is defined as *Internal Work* or *Strain Energy* of the system, which is symbolised by $U$. Amount of strain energy is numerically equal to the external work done on the members.

As loading is done within proportional limit, as soon as load is released, the system also will lose energy and will come back to original position. This property of an elastic material to absorb and release energy with change in loading is called *Resilience*.

Here total strain energy of the system

$$U = \frac{1}{2} P \cdot \delta = \frac{1}{2} (\sigma A)(\varepsilon \cdot l) = \frac{1}{2} (\sigma \cdot \varepsilon)(A \cdot l)$$ \hfill (7.3)

Therefore strain energy per unit volume

$$u = \frac{U}{Al} = \frac{1}{2} \sigma \varepsilon = \frac{\sigma^2}{2E}$$ \hfill (7.4)

Another very important parameter in this context is *Proof resilience*. It is the maximum strain energy stored in a body before yielding occurs. Strain energy will be maximum when the body will be stressed up to elastic limit. And proof resilience per unit volume is known as *Modulus of Resilience*.

$$\therefore u_r = \frac{\sigma_p^2}{2E}$$ \hfill (7.5)

Sometimes $\sigma_p$ stands for yield stress or proof stress, as the case may be.
7.14 THERMAL STRESS

Any engineering material, when subjected to change in temperature exhibits expansion in temperature rise and contraction in temperature fall. This change in temperature is often termed as Thermal Loading.

A structural member of length $l$ if subjected to thermal loading of $\Delta T$, can expand or contract by an amount, $\delta = l \alpha \Delta T$, where $\alpha$ = thermal coefficient of material, when free expansion or contraction is allowed. In non-restrained system no stress will be developed, though there presents thermal loading.

If free movement of the member is restricted partially or fully by somehow or other, some amount of reactive force will be generated within the member, which will give birth to a reactive stress, termed as thermal stress.

Select the best alternative (s):

1. The impact strength of a material is an index of its
   (a) toughness  (b) tensile strength  (c) capability of being cold worked
   (d) hardness  (e) fatigue strength
2. The property of a material which allows it to be drawn into a smaller section is called
   (a) plasticity  (b) elasticity  (c) ductility  (d) malleability
3. The loss of strength in compression due to overloading is known as
   (a) hysteresis  (b) relaxation  (c) creep  (d) resilience
   (e) bauschinger effect
4. The maximum strain energy that can be stored in a body is known as
   (a) impact energy  (b) resilience  (c) proof resilience
   (d) modulus of resilience  (e) toughness
5. The total strain energy stored in a body is termed as
   (a) resilience  (b) proof resilience  (c) modulus of resilience
   (d) toughness
6. Proof resilience per unit volume of a material is known as
   (a) resilience  (b) proof resilience  (c) modulus of resilience
   (d) toughness
7. The Figure 7.1 shows the stress-strain diagram for mild steel. The elastic limit, upper yield point, lower yield point and proportional limit are represented by
   (a) $A, B, C, D$  (b) $A, C, D, B$  (c) $B, C, D, A$
   (d) $C, B, D, A$  (e) $B, C, A, D$
8. Proof stress is
   (a) stress corresponding to proportional limit
   (b) stress causing materials to break
   (c) stress causing a specific permanent deformation usually 0.1% or 0.2%.
   (d) not related with engineering
9. Thermal strain caused in the material of a composite body due to change in temperature will be
   (a) same nature    (b) opposite nature   (c) same magnitude   (d) none of above

10. True stress ($\sigma$) is related to simple stress ($\sigma$) and strain ($\varepsilon$) by
    (a) $\sigma = \sigma (1 - \varepsilon)$    (b) $\sigma = \sigma (1 + \varepsilon)$  
    (c) $\sigma = \sigma \varepsilon$  (d) $\sigma = \frac{\sigma}{\varepsilon}$

11. Bulk modulus $K$ in terms of modulus of elasticity ($E$) and Poisson’s ratio ($\mu$) is given as equal to
    (a) $E / 3(1 - 2\mu)$    (b) $E(1 - 2\mu)$  
    (c) $3E(1 - 2\mu)$  
    (d) $E(1 + 2\mu) / 3$  
    (e) $E(1 - 3\mu) / 3$

12. The energy absorbed by a body, when it is strained within the elastic limit, is known as
    (a) strain energy    (b) resilience    (c) proof resilience
    (d) modulus of resilience    (e) toughness

13. Value of factor of safety is
    (a) greater than 1    (b) less than 1
    (c) zero    (d) none of above

14. Hooke’s Law is truly valid up to
    (a) elastic limit    (b) proportional limit
    (c) plastic limit    (d) fatigue limit

15. In ductile material nominal breaking stress is
    (a) lower than true breaking stress    (b) equals with true breaking stress
    (c) higher than true breaking stress   (d) none of the above

16. In ductile material ultimate stress is
    (a) higher than true breaking stress    (b) lower than nominal breaking stress
    (c) higher than nominal breaking stress but lower than true breaking stress
    (d) higher than true breaking stress but lower than nominal breaking stress

17. In a brass specimen subjected to tension, which of the following can be obtained in stress-strain diagram?
   (a) upper yield stress    (b) lower yield stress    (c) plastic stress    (d) proof stress

18. Thermal change of length of a metal is related to its thermal coefficient
    (a) inversely proportional    (b) directly proportional
    (c) directly square proportional    (d) none of the above

19. Thermal strain of a body does not depend on
    (a) length    (b) thermal coefficient    (c) change in temperature
    (d) none of the above

20. In brittle material, normally, breaking stress is
    (a) higher than ultimate stress    (b) lower than ultimate stress
    (c) equals with ultimate stress    (d) none of the above
A steel bar of 25mm diameter was tested in tension and results were recorded as, limit of proportionality = 196.32kN, load at yield = 218.13kN, ultimate load = 278.20 kN. The elongation measured over a gauge length of 100mm was 0.189mm at proportionality limit, length of the bar between gauge marks after fracture was 112.62mm and minimum diameter was 23.64mm. Compute stress in the specimen at various stages, Young’s modulus, % elongation and % contraction. Determine permissible stress in the material for a safety factor of 1.85.

**Solution**  
Initial c/s area of the bar = \( \frac{\pi}{4} \times (25)^2 = 490.874 \text{ mm}^2 \)

\[ \therefore \text{Stress at proportionality limit} = \frac{196.32 \times 10^3}{490.874} = 399.94 \text{ MPa} \]

\[ \therefore \text{Strain at proportionality limit} = \frac{0.189}{100} = 1.89 \times 10^{-3} \]

\[ \therefore \text{Young’s modulus} = \frac{399.94}{1.89 \times 10^{-3}} = 2.116 \times 10^5 \text{ MPa} \]

\[ \therefore \text{Stress at yield point} = \frac{218.13 \times 10^3}{490.874} = 444.37 \text{ MPa} \]

\[ \therefore \text{Ultimate stress} = \frac{278.20 \times 10^3}{490.874} = 566.74 \text{ MPa} \]

\[ \therefore \text{Final c/s area at fracture} = \frac{\pi}{4} \times (23.64)^2 = 438.919 \text{ mm}^2 \]

\[ \% \text{ elongation} = \frac{112.62 - 100}{100} \times 100 = 12.62 \]

\[ \% \text{ contraction in area} = \frac{490.874 - 438.919}{490.874} \times 100 = 10.584 \]

Permissible or working or allowable stress = \( \frac{\text{yield stress}}{\text{factor of safety}} \)

\[ = \frac{444.37}{1.85} = 240.2 \text{ MPa} \]
**Example 2**

Steel railroad, 10m long, is laid with a clearance of 3mm at 15°C. At what temperature will the rails just touch? What stress will be induced in the rails at that temperature, if there were no initial clearance, while $a = 117\text{mm/m°C}$ and $E = 200\text{ GPa}$.

**Solution**

Let the desired temperature = $T^\circ \text{C}$

Now, free thermal elongation from 15°C to $T^\circ \text{C}$ = $10 \times 10^3 \times 11.7 \times 10^{-6} \times (T - 15)$

As per condition provided,

$$0.117(T - 15) = 3$$

$$T = 40.64 \, ^\circ \text{C}$$

At no clearance condition, induced strain will be $$\frac{3}{10 \times 10^6} = 3 \times 10^{-4}$$

and induced stress = $$3 \times 10^{-4} \times 200 \times 10^3 \text{ MPa} = 60\text{MPa}$$

**Example 3**

A steel rod 3ft long with c/s area of 0.25 inch$^2$ is stretched between two fixed points. The tensile force is 1200 lb at 40°F. Using $E = 29 \times 10^6$ psi and $a = 6.5 \times 10^{-6}$ inch/inch/°F Calculate $(a)$ the temperature at which the stress in the bar will be 10 ksi, $(b)$ the temperature at which the stress will be zero.

**Solution**

Initial stress in the rod = $$\frac{1200}{0.25} = 4,800 \text{ psi}$$

$(a)$ Required stress is to be developed = 10,000 psi

∴ Additional stress to be developed = 10,000 – 4,800 = 5,200 psi

This additional stress will be generated due to rise in temperature, say at $T^\circ \text{F}$

∴ Strain corresponding to additional stress = $$\frac{5,200}{29 \times 10^6} = 1.793 \times 10^{-4}$$

∴ Elongation due to above strain = $(1.793 \times 10^{-4})$ inch

= $6.45 \times 10^{-4}$ inch

Hence, $3 \times 12 \times 6.5 \times 10^{-6} (T - 40) = 6.45 \times 10^{-3}$

⇒ $T = 67.564 \, ^\circ \text{F}$

$(b)$ As stress is to be zero, temperature of the system will have to be reduced.

Let that temperature be $T_i \, ^\circ \text{F}$.

Now, strain corresponding to initial stress = $$\frac{4,800}{29 \times 10^6} = 1.655 \times 10^{-4}$$

Thermal strain due to reduction of temperature = $6.5 \times 10^{-6} \times (40 - T_i)$

Here, $6.5 \times 10^{-6} \times (40 - T_i) = 1.655 \times 10^{-4}$

⇒ $T_i = 14.538 \, ^\circ \text{F}$
EXAMPLE 4

A bronze bar, 3m long with a c/s area of 320mm$^2$ is placed between two rigid walls. At $-20^\circ$C, the gap between bar and wall is 2.5mm. Find temperature at which compressive stress in the bar will be 35MPa. Take $a=18\times10^{-6}$m/m$^\circ$C and $E=80$GPa.

**SOLUTION**  
Strain corresponding to 35MPa stress  
\[
\varepsilon = \frac{35}{80 \times 10^3} = 4.375 \times 10^{-4}
\]

∴ Elongation due to above strain  
\[
4.375 \times 10^{-4} \times 3000 = 1.3125 \text{mm}
\]

To generate above compressive stress, total elongation to be compensated by thermal rise will be

\[
(1.3125 + 2.5) \text{ mm} = 3.8125 \text{ mm}
\]

and let the final temperature be $T^\circ$C.

Now,

\[
3 \times 10^3 \times 18 \times 10^{-6} \times (T + 20) = 3.8125
\]

⇒ \[ T = 50.6^\circ \text{C}. \]

EXAMPLE 5

Calculate increase in stress for each segment of the compound bar, if temperature increases by 100$^\circ$F. Assume unyielding supports and bar is suitably braced against buckling. If supports yield by 0.01 inch, compute stresses.

**SOLUTION**  
If free thermal elongation would be allowed, it will be for aluminium

\[
(10 \times 12.8 \times 10^{-6} \times 100) = 0.0128 \text{ inch}
\]

for steel  

\[
(15 \times 6.5 \times 10^{-6} \times 100) = 9.75 \times 10^{-3} \text{ inch}
\]

and total  

\[
(0.0128 + 9.75 \times 10^{-3}) = 0.02255 \text{ inch}
\]

As the supports are unyielding, to resist this elongation some mechanical stress will be developed and corresponding contraction will be:

\[
\frac{\sigma_a}{E_a} \times L_a = \left( \frac{\sigma_a}{10 \times 10^6} \times 10 \right) \text{ inch}
\]

for aluminium

\[
\frac{\sigma_s}{E_s} \times L_s = \left( \frac{\sigma_s}{29 \times 10^6} \times 15 \right) \text{ inch}
\]

for steel

\[
\frac{\sigma_a}{E_a} \times L_a = \left( \frac{\sigma_a}{10 \times 10^6} \times 10 \right) \text{ inch}
\]

To satisfy compatibility for deformation,

\[
\left( \frac{\sigma_a}{10 \times 10^6} \times 10 \right) + \left( \frac{\sigma_s}{29 \times 10^6} \times 15 \right) = 0.02255
\]

⇒ \[ \sigma_a + 0.5172\sigma_s = 22550 \]

Another equation of compatibility,

\[
\sigma_a A_a = \sigma_s A_s
\]

⇒ \[ \sigma_a \times 2.0 = \sigma_s \times 1.5 \]

⇒ \[ \sigma_a = 0.75\sigma_s \]
Solving equations (3) and (4), \( \sigma_u = 13,346.35 \text{ psi}, \quad \sigma_s = 17,795.14 \text{ psi} \)

If the supports yield, equation of compatibility will be

\[
\frac{\sigma_u}{10 \times 10^6} \times 10 + \frac{\sigma_s}{29 \times 10^6} \times 15 = 0.02255 - 0.01
\]

\[\Rightarrow \quad \sigma_u + 0.5172\sigma_s = 12550 \quad \text{...(5)}\]

Solving equations (5) and (4),

\[
\sigma_u = 7,427.79 \text{ psi} \\
\sigma_s = 9,903.72 \text{ psi}
\]

**Example 6**

At 80°C a steel tire, 12mm thick and 90mm wide is to be shrunk fit onto a locomotive wheel, 2m in diameter, just fits over the wheel which is at a temperature of 25°C. Determine contact pressure between tire and wheel at 25°C. \( \alpha = 11.76 \times 10^{-6} \text{ m/m/ºC} \) and \( E = 200 \text{ GPa} \).

**Solution**

Let diameter of tire at 25°C = \( d \) and of wheel = \( D \)

Considering condition of compatibility,

\[
\pi \times D = \pi \times d \left[ 1 + \alpha (80 - 25) \right]
\]

\[= \pi \times d [11.76 \times 10^{-6} \times 55] \]

\[\Rightarrow \quad \frac{D}{d} = 1.0006468 \quad \text{...(1)}\]

Circumferential strain in the wheel

\[
\varepsilon = \frac{\pi D - \pi d}{\pi d} = \left( \frac{D}{d} - 1 \right)
\]

\[= 0.0006468\]

Corresponding stress in the wheel = (0.0006468 \times 200 \times 10^3) MPa

\[= 129.36 \text{MPa}\]

**Example 7**

In the adjoining figure, bar \( ABC \) is initially horizontal and vertical rods are stress-free. Determine stress in the aluminium rod, if temperature of the steel rod is decreased by 40°C.

\( E_s = 200 \times 10^9 \text{ N/m}^2, \quad E_a = 70 \times 10^9 \text{ N/m}^2, \)

\( \alpha_s = 11.7 \mu \text{m/m/ºC}, \quad \alpha_a = 23 \mu \text{m/m/ºC}, \)

**Solution**

From free-body diagram of the bar, taking moment at \( B \), the condition of equilibrium will be,

\[R_d \times 0.6 = R_c \times 1.2\]

\[\Rightarrow \quad (\sigma_u \times A_u) \times 0.6 = (\sigma_s \times A_s) \times 1.2\]
Stress-Strain Diagram and Strength Parameters

\[ \Rightarrow \quad \sigma_{st} \times 300 \times 0.6 = \sigma_{al} \times 1200 \times 1.2 \]

\[ \Rightarrow \quad \sigma_{st} = 8 \sigma_{al} \quad \text{...(1)} \]

Due to reduction of temperature, strain in the steel rod

\[ = 11.7 \times 10^{-6} \times 40 \]
\[ = 4.68 \times 10^{-4} \]

Corresponding stress steel rod (\( \sigma_{st} \))

\[ = (4.68 \times 10^{-4} \times 200 \times 10^9 \times 10^{-6}) \]
\[ = 93.6 \text{MPa} \]

From (1),
\[ \sigma_{st} = \frac{93.6}{8} = 11.7 \text{MPa} \]

**Example 8**

A steel tube of 24mm external and 14mm internal diameters encloses a copper rod of 12mm diameter. The assembly is held rigidly at both ends at 22°C. Compute (i) stresses at 122°C, (ii) the maximum temperature the assembly can withstand. Assume

\[ E_s = 220 \text{GPa}, \]
\[ \alpha_s = 11 \times 10^{-6} / ^\circ \text{C}, \]
\[ \alpha_c = 18 \times 10^{-6} / ^\circ \text{C}, \]
\[ (\sigma_s)_{max} = 230 \text{MPa}, (\sigma_c)_{max} = 115 \text{MPa}, \]

**Solution**

As \( \alpha_s > \alpha_c \), copper rod will expand more than steel tube, if free expansion is allowed. As both ends of assembly are fastened rigidly, free expansion will not be permitted in either rod or tube. A thermal compromise will be happened. By virtue of it, free expansion of copper rod will be reduced and free expansion of steel tube will be increased. At this level, a mechanically induced compressive force will act at copper rod and a tensile force will act at steel tube. Both of these forces are equal in magnitude.

Hence,

\[ L \cdot \alpha_s \cdot \Delta T - \frac{P_c \cdot L}{A_c \cdot E_c} = L \cdot \alpha_s \cdot \Delta T + \frac{P_s \cdot L}{A_s \cdot E_s} \quad \text{...(1)} \]

\[ \Rightarrow \quad (\alpha_s - \alpha_c) \Delta T = \frac{\sigma_s}{E_c} + \frac{\sigma_c}{E_s} \quad \text{...(2)} \]

\[ 7 \times 10^{-6} \times (122 - 22) = \left( \frac{\sigma_s}{110} + \frac{\sigma_c}{220} \right) \times \frac{1}{10^5} \]

\[ \Rightarrow \quad \sigma_s + 0.5 \sigma_c = 77 \quad \text{...(3)} \]

From equation of compatibility,

\[ P_c = P_s \]

\[ \Rightarrow \quad \sigma_c \cdot A_c = \sigma_s \cdot A_s \]
\[
\sigma_c = \sigma_s \frac{A_s}{A}
\]
\[
= \sigma_s \frac{24^2 - 14^2}{12^2} = 2.639 \sigma_s
\]

Solving equations (3) and (4)

\[
\begin{align*}
\sigma_c &= 64.73 \text{ MPa} \\
\sigma_s &= 24.53 \text{ MPa}
\end{align*}
\]

To find maximum withstandable temperature \( T \), we are to substitute maximum permissible stresses in equation (2)

\[
7 \times 10^{-6} \times (T - 22) = \left[ \frac{\sigma_c}{110} + \frac{\sigma_s}{220} \right] \times \frac{1}{10^3}
\]

If \( (\sigma_c)_{\text{max}} = 230 \text{ MPa} \), according to equation (4), \( \sigma_s = 87.154 \text{ MPa} \)

If \( (\sigma_s)_{\text{max}} = 115 \text{ MPa} \), according to equation (4), \( \sigma_s = 303.485 \text{ MPa} \), which is higher than \( (\sigma_c)_{\text{max}} \). So, maximum allowable stress in copper and steel will be 230 MPa and 87.154 MPa respectively.

Substituting in equation (5),

\[
7 \times 10^{-6} \times (T - 22) = \left( \frac{87.154}{110} + \frac{230}{220} \right) \times \frac{1}{10^3}
\]

\[
\Rightarrow T = 284.54^\circ \text{C}
\]

**Example 9**

A bar of uniform c/s \( A \) and length \( L \) hangs vertically, subjected to its own weight. Prove that the strain energy stored within the bar is \( U = \frac{Ap^2L^3}{6E} \).

**Solution** Let us take a section \( x \) distance from bottom, of thickness \( dx \). The elongation of length \( dx \) be \( d\delta \).

\[
\therefore \quad \text{Strain in length } dx, \quad \varepsilon_x = \frac{d\delta}{dx}
\]

Stress in length \( dx \),

\[
\sigma_x = \frac{W_x}{A} = \rho \frac{Ax}{A} = \rho x
\]

Now,

\[
E = \frac{\sigma_x}{\varepsilon_x} = \frac{\rho x dx}{d\delta}
\]

\[
\Rightarrow d\delta = \frac{\rho x dx}{E}
\]

Strain energy stored in \( dx \)

\[
dU = \text{average weight } \times \text{ elongation of } dx
\]

\[
= \left( \frac{1}{2} W_x \right) \frac{\rho x dx}{E}
\]

\[
= \frac{1}{2} \rho x \left( \frac{\rho x dx}{E} \right) = \frac{Ap^2}{2E} x^2 dx
\]
In the adjacent figure, load is allowed to drop on the collar from a height $h$, find the expression of stress induced in the rod due to impact.

**Solution**

Strain in the bar

$$\frac{\delta L}{L} = \frac{\sigma}{E}$$

$$\Rightarrow \delta L = \frac{\sigma}{E} \times L \quad \text{...(1)}$$

Work done by the load

$$W = W(h + \delta L) \quad \text{...(2)}$$

Strain energy stored by the rod

$$\frac{\sigma^2}{2E} (AL) \quad \text{...(3)}$$

Equating (2) and (3) following condition of equilibrium,

$$W(h + \delta L) = \frac{\sigma^2}{2E} (AL) \quad \text{...(4)}$$

Substituting the expression of (1) into (4), we have,

$$\sigma^2 - \left( \frac{2W}{A} \right) \sigma - \frac{2WEh}{AL} = 0$$

Solving,

$$\sigma = \frac{W}{A} \left( 1 + \sqrt{1 + \frac{2AEh}{WL}} \right) \quad \text{...(5)}$$

Sometimes, $W \left( 1 + \sqrt{1 + \frac{2AEh}{WL}} \right)$ term is designated as equivalent static load, i.e.,

$$W_e = W \left( 1 + \sqrt{1 + \frac{2AEh}{NL}} \right)$$

(i) if $\delta L \ll h$, from equation (4),

$$Wh = \frac{\sigma^2}{2E} (AL)$$

$$\Rightarrow \sigma = \sqrt{\frac{2EhW}{AL}} \quad \text{...(6)}$$
(ii) If $h = 0$, from equation (4)

$$\sigma = \frac{2W}{A} \quad \ldots \quad (7)$$

**Example 11**

Find the strain energy of these two members, loaded and shown in the figure.

**Solution** Strain energy (S.E) of a bar subjected to loading $P$

$$S.E = \frac{1}{2} P \cdot \delta = \frac{1}{2} \frac{P^2 l}{AE}.$$  

For the first member, S.E

$$S.E = \frac{1}{2} \frac{P^2 \left(\frac{3l}{8}\right)}{2 \frac{\pi}{4}(2d)^2 \cdot E} + \frac{1}{2} \frac{P^2 \left(\frac{l}{4}\right)}{2 \frac{\pi}{4}(2d)^2 \cdot E} + \frac{1}{2} \frac{P^2 \left(\frac{3l}{8}\right)}{2 \frac{\pi}{4}(2d)^2 \cdot E}$$

$$= \frac{1}{2} \frac{4 \times \frac{P^2 l}{2 \pi d^2 E} \left(\frac{3}{8 \times 4} + \frac{1}{4} + \frac{3}{8 \times 4}\right)}{16}$$

$$= \frac{2P^2 l}{\pi d^2 E} \times \frac{7}{16} = \frac{7P^2 l}{8 \pi d^2 E}$$

For the second member, S.E

$$S.E = \frac{1}{2} \frac{P^2 \left(\frac{9l}{20}\right)}{2 \frac{\pi}{4}(3d)^2 \cdot E} + \frac{1}{2} \frac{P^2 \left(\frac{l}{10}\right)}{2 \frac{\pi}{4}(3d)^2 \cdot E} + \frac{1}{2} \frac{P^2 \left(\frac{9l}{20}\right)}{2 \frac{\pi}{4}(3d)^2 \cdot E}$$

$$= \frac{1}{2} \frac{4 \times \frac{P^2 l}{2 \pi d^2 E} \left(\frac{9}{20 \times 9} + \frac{1}{10} + \frac{9}{20 \times 9}\right)}{5}$$

$$= \frac{2P^2 l}{\pi d^2 E} \times \frac{1}{5} = \frac{2P^2 l}{5 \pi d^2 E}.$$

**Exercise**

1. A mass of 200 kg falls through a height of 500 mm on a concrete column of $300 \times 400$ mm section. Determine the maximum stress and deformation in the 4.0m long column, considering modulus of elasticity of concrete 20.0GPa.

**Ans.** $[-9.06\text{MPa}, -1.81\text{mm}]$
2. A rod is 3m long at a temperature of 15°C. Find the expansion of the rod, when the temperature is raised to 95°C. If this expansion is prevented, find the stress induced in the material of the rod. Take $E = 1 \times 10^5$ N/mm$^2$ and $\alpha = 0.000012$ per degree centigrade. \textbf{Ans.} [0.288cm, 96N/mm$^2$]

3. A steel rod 5cm diameter and 6m long is connected to two grips and the rod is maintained at a temperature of 100°C. Determine the stress and pull exerted when the temperature falls to 20°C if (i) the ends do not yield, and (ii) the ends yield by 0.15cm. Take $E = 200$GPa, $\alpha = 12\times10^{-6} / ^\circ C$.

4. Compute the maximum force a 200mm long compound bar comprising a copper rod of 18 mm diameter enclosed in a mild steel tube of 200mm inner and 32 mm outer diameters can sustain. Assume Young’s modulii for copper and steel to be 120GPa and 195GPa, respectively. What will be the reduction in the maximum load, if the bar temperature rises by 40K? Find the reduction in the strength when the temperature falls by 40K. Allowable stresses are 80MPa and 140MPa in copper and steel, respectively; $\alpha = 18 \times 10^{-6} K^{-1}$ for copper and $12 \times 10^{-6} K^{-1}$ for steel.

5. A 3.5m long steel column of cross-sectional area 5000mm$^2$ is subjected to a load of 1.6 MN. Determine the safety factor for the column, if the yield stress of steel is 550MPa. Determine the allowable load on the column, if the deformation of the column should not exceed 5mm.

6. A compound bar comprises a 12.5mm diameter aluminum rod and a copper tube of 14.5mm inner and 25mm outer diameters. If the Young’s modulii of aluminium and copper are 80GP and 120GPa, respectively, determine the stress in the assembly when subject to (i) a temperature rise of 95K, and (ii) a temperature fall of 35K; $\alpha = 14.6 \times 10^{-6} K^{-1}$ for aluminium and $16.8 \times 10^{-6} K^{-1}$ for copper.

7. A steel rod of 20mm diameter passes centrally through a copper tube of 40mm external diameter and 30mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly home on the projected parts of the rod. If the temperature of the assembly is raised by 60°C, calculate the stresses developed in copper and steel. Take $E$ for steel and copper as 200GN/m$^2$ and 100GN/m$^2$ and $\alpha$ for steel and copper as $12 \times 10^{-6}$ per °C and $18 \times 10^{-6}$ per °C. \textbf{Ans.} [16.23, 28.4 N / mm$^2$]