1.1 The Changing Scenario of Business

Until recently, it was felt that the analysis of events that take place in business needs only a good sense and a sound logical reasoning. However, the complexity of modern business environment makes the process of decision-making more difficult. Thus, the decision-maker cannot completely rely upon the commonsense observations and obtained business experience. Recent trends in the development of business certainly have convinced all of us that there is an urgent need for a better understanding of the subject matter. No doubt, this improved understanding can be brought about only through a scientific approach to the complex real world happenings in business. In the process, one must bring the date old verbal business related theories within the framework of technical competence that makes it possible to use the up-to-date mathematical and statistical techniques. In fact business is basically a quantitative subject in the sense that most of the variables like prices, income, employment, profit etc., that we use in business are always measured and stated in quantitative terms. Business theories often deal with the analysis of relationships between variables. When such relationships are laid down in a specific format they form business theories or otherwise called business principles. For example, the inverse relationship between price charged and the quantity demanded is popularly known as the law of demand in business. Similarly, the relationship between inputs and outputs solely determined by the state of technical knowledge of the vintage concerned is called the production function. In all these relationships the involved variables are measured and stated in quantitative terms. Moreover, to depict the exact form of the said relationships, linear or otherwise, it is handy to use both mathematical and statistical techniques in various combinations.

The scientific management revolution of the early 1900’s initiated by Frederic W. Taylor provided the foundation for the use of quantitative methods in business management. The recent development in the use of statistical data in making business decisions has created some new and powerful tools of analysis. The pressure of more complex business problems and large-scale operation of business has stimulated research in the development sophisticated tools. Many fields of knowledge have contributed to this improvement in business decision-making. The use of more objective methods and more accurate information improves the understanding of business problems. The increasing use of statistical methods has provided more and more precise information. In recent years a great deal of experimentation has been carried out to bring out improved method of understanding the business. During World War-II, the methods used by
Mathematical Methods for Management

scientists were applied with outstanding success to certain military problems, such as mining the enemy’s water, searching out enemy’s ship, etc. Two developments that occurred during post war II period led to the growth and use of quantitative methods in non-military applications. Continued research resulted in numerous methodical developments. Probably the most significant development was the discovery of simplex method for solving linear programming problems by Ghaige Dantzig in 1947. Subsequently, many methodological developments followed. In 1957 Churchman, Ackoff, and Arnoff published the first book on operation research. Business organizations are now successfully apply these techniques to problems like inventory fluctuations, the product mix that maximizes profit, shipments of goods from origin’s to different destinations etc. Mathematics, statistics, physics, economics, and engineering are some of the fields in which operations research methods are used effectively. Since the variables under consideration are expressed in quantities it is often called quantitative technique as well.

1.2 Mathematics and Management Decisions

Nowadays, the branch of knowledge that uses mathematical techniques specially in economic theorizing is popularly known as “Mathematical Economics”. In explaining the economic theory the use of mathematics makes the theory not only simple but also more precise and exact. In fact in mathematical economics we transform the economic theory into a compact and precise mathematical form by using appropriate mathematical functional form. For example, the law of demand tells us, that when other thing does not change the price and quantity demanded are inversely related. As a first approximation to this demand law, economists often use linear equations of the type \( q = a + bp; \ a > 0, \ b < 0 \) to make the analysis simple. Similarly, to represent consumer's preference in the theory of consumption we often use convex indifference curves. Often economic theories are expressed in mathematical terms to obtain their important characteristic results. However, it should be remembered that mathematical economics is certainly not a separate branch of knowledge by itself. In fact, the mathematical approach is common to all fields in economics, like microeconomics, macroeconomics, public finance etc. Hence mathematical economics is considered as a scientific approach, which can be used in almost all branches of economics. The only difference now is that, in mathematical economics we use mathematical language instead of the verbal language in explaining the concerned economic theory. However, it should not be mistaken that in mathematical economics the economic theory is simply transformed into its mathematical form. The very purpose of such a transformation is not only to make the theory easy but also to arrive at certain interesting characteristic results. For example, after transforming both demand and supply functions in its simple linear mathematical form we can easily calculate both the equilibrium price and the quantity. Similarly, we can calculate the appropriate tax rate that gives maximum tax collection to the government. It is important to note that such typical questions can be answered more precisely only by using mathematics. Therefore mathematical economics can always be considered as complementary rather than competitive in economic analysis.

1.3 Advantages of Using Mathematics in Business

1. The mathematical language by nature is concise and precise. Hence, by using mathematics we can restate the business theory in a more compact form like the one
stated above in the case of the demand law. In it the involved relationship is simple and self-explanatory in its mathematical form.

2. The mathematical simplicity enhances the precision of analysis like the calculation of equilibrium price, equilibrium quantity, price elasticity of demand etc.

3. The mathematical approach can have always the advantage of using the ever-growing unlimited amount of tools and theorems in pure mathematics for their advantage. The use of Euler’s mathematical theorem in business in explaining the distribution of income among the factors of production is the classical example for such an advantage.

4. Once a certain specific mathematical relationship is obtained, the business manager can deduce more useful new propositions.

5. The biggest advantage of mathematical science is its ability to handle large number of variables at a given point of time. For example, in the theory of consumption especially in indifference curve analysis, at the most we can handle only two commodities, one along the x-axis and one along y-axis. But in reality, our consumption basket contains a large number of commodities. Mathematical science can handle this situation by increasing the commodity space which can accommodate any number of commodities in getting the extended equi-marginal principle.

1.4 Disadvantages of Using Mathematics in Business

1. In certain sections of business, we come across variables like the tastes, preferences etc. Since these variables are qualitative in nature we often find it difficult to use quantitative mathematical techniques.

2. The most common criticism leveled against mathematical interpretation of business problem is about its abstract and unrealistic nature. The abstract and unrealistic results derived by using mathematical methods are mainly due to unrealistic assumptions that have been made in the beginning about the concerned theory. Moreover, such unrealistic and abstract results are often found in non-mathematical theoretical treatment as well. Therefore, wherever the assumptions of the theory are more realistic, less will be the abstract nature of the theory both in mathematical and in non-mathematical presentations.

1.5 Problem Solving and Decision-Making

Decision-making is the term normally associated with the following steps:

- Identify and define the problem.
- Determine the alternative solutions.
- Determine the criterion that will be used to evaluate the alternatives.
- Evaluate the alternatives.
- Choose the best alternative.

Let us consider the following illustrative example for the decision-making process.

**Problem defined:** Assume that you are in a well-paid job in Weprow, a leading IT industry in Bangalore, and would like to move to some other company. Then you define the alternatives as follows:
• Move to Infosys
• Move to Micro System
• Network Solution
• System Management.

Once the four alternatives are defined, the next job is to define criteria’s for selecting one among the four. Let the highest starting pay is the first criterion. Now the job of selection is simply the highest starting paying job. This type of problems is called single criterion problems.

Suppose in addition to the highest starting salary, you also want to have nearest work location and potential for promotional avenues as additional criterions. Now since your problem involves three criterions it is called multi-criterion problem. Evaluating each alternative with added criterions is a difficult job because evaluations are based on subjective factors which are difficult to quantify. Let us rate the added criterions as poor, average, good and excellent. Now let the data relating to our problem are as shown below:

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Starting Salary</th>
<th>Potential Avenues for Promotions</th>
<th>Job Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rs. 35,000</td>
<td>average</td>
<td>average</td>
</tr>
<tr>
<td>2</td>
<td>Rs. 40,000</td>
<td>good</td>
<td>good</td>
</tr>
<tr>
<td>3</td>
<td>Rs. 42,000</td>
<td>excellent</td>
<td>excellent</td>
</tr>
<tr>
<td>4</td>
<td>Rs. 50,000</td>
<td>average</td>
<td>good</td>
</tr>
</tbody>
</table>

Now with this added scaling information we are better placed in decision-making. Alternative 3 seems to be the best decision under the given circumstances.

Once the decision is made to go for alternative 3, the decision is implemented and the result is evaluated.
Define the problem

Identify the alternatives

Define the selection criteria

Evaluate the alternatives

Choose an alternative

**Fig. 1.2**

Figure 1.1 shows the analysis phase of the decision-making problem. In any decision making problem one will face the both qualitative and quantitative judgements. The qualitative judgement decision is often based on the value judgement experience of the decision-maker. It normally includes the decision-maker’s intuitive “feel” of the problem and hence based on considered as an art rather then science. If the decision-maker is well experienced in the related field, then one must give due respect to the qualitative decision and accept his judgement. Alternatively, if the decision-maker is with little experience in the field and the problem is more complex, then one must opt for quantitative analysis. Under the circumstances, using one or more quantitative methods, the analysts will recommend the appropriate optimum decision.

Although the skill in the qualitative part of the problem is based on experience, the quantitative skill goes beyond one’s own comprehension. One can acquire this skill only by learning management science. In fact these two approaches are complementary in optimum decision-making. The following chart [Fig. 1.3] summarises the entire process of decision making.

**EXERCISES**

1. Discuss the importance of quantitative methods in business.
2. Do you think that quantitative techniques have a definite role to play in every future decision-making? Substantiate your answer with suitable illustrations.
3. Discuss fully the limitations of quantitative techniques in business.
4. “Statistics is the straw out of which I, like every other economists, make my bricks”. Explain.
5. With illustrations examine the role of mathematical and statistical methods in business.
6. Discuss fully the limitations of statistics in business.
7. Discuss in detail the importance of statistics with special reference to business.
8. Explain the usefulness of statistics in business analysis and planning in particular.
10. “Statistics are like clay of which you can make either a God or Devil as you please”. Give your comment on the above statement.
11. Describe the role of both mathematical and statistical methods in business.
12. “Quantitative analysis has considerable accomplishments to its credits but it is not a panacea, and hence is in occasional danger of living oversold.” Discuss.
13. Describe the steps involved in problem formulation and problem solving.
2 Chapter

THE THEORY OF SETS

2.1 Introduction
In our day-to-day life, we all use the concept ‘set’ invariably in all walks of life. In all cases by set we simply mean the collection of some well-defined objects like the set of teachers in the college, the set of planets in our solar system, the set of Economics text books in the University library etc. In all these examples, the said objects have been grouped together and viewed as a single entity. This idea of grouping objects together gives rise to a mathematical concept often called the set. Thus, the objects of a given set may be anything, as we please, expect the fact that there should not be any room for doubt as whether the given object belongs to the set under reference or not.

2.2 Definition of a Set
These sets are often denoted by capital letters A, B, C, etc. The elements of a set are written within a pair of braces {}. The following are the set of some well-defined objects.

A = {The set of colours in the Indian National Flag}
B = {Bangalore, Mumbai, Chennai, Delhi}
C = {1, 2, 3, 4, 5, 6, 7}
D = {a, b, c, d, e,}
E = {The set of all even numbers}

In general if ‘A’ is the set and ‘x’ is a member of the set, then we simply say that ‘x’ belongs to the set A. To make the representation simple, we often use the symbol ‘∈’ to mean ‘belongs to’ and write the above statement simply as \( x \in A \). If x is not a member of the given set A, then to mean ‘does not belongs to’ we use the symbol ‘\( \notin \)’ and write the same as \( x \notin A \).

Example 1: If A = {1, 2, 3} then since 2 is a member of the set A we would say that 2 ∈ A. Further, since 4 is not a member of the set A we can also say that 4 \( \notin \) A.

The elements of a given set may either be written completely by listing out all its members or by writing some common characteristics to represent them all.

Example 2: A = {saffron, white, green} or
A = {The set of all colours in the Indian National Flag}. 
In a set, the order in which the elements are placed does not matter at all. For example, the set \( A = \{1, 2, 3, 4, 5\} \) and the set \( B = \{5, 3, 4, 2, 1\} \) are equal to one another in all respects, though the order of placement of the individual elements are different. Similarly, we do not count a certain given member more than once in a given set though there are repetitions. By this we mean that the sets

\[ A = \{1, 2, 3\} \text{ and } B = \{1, 2, 2, 3, 3\} \]

are equal to one another in all respects.

**Finite and Infinite Sets:** A set, which has finite number of elements is called a finite set. Similarly, the set that contains infinite number of elements is called an infinite set.

**Example 3:** \( A = \{a, b, c, d\} \) is called a finite set because it contains four finite number of elements in it.

**Example 4:** Similarly, the set \( B = \{\text{the set of all positive integers}\} \) is called an infinite set because it contains infinite number of positive integers in it.

**Notations in Sets:** To describe a given set, in general we often use certain specialized type of notations.

**Example 5:** Let \( B = \{x/x \text{ is an integer}\} \). We read this statement as ‘\( B \) is the set of all \( x \)’s such that \( x \) is an integer’. Alternatively, by this notation we simply mean that \( B \) is the set of all integers. i.e., \( B = \{\ldots -3, -2, -1, 0, 1, 2, 3\ldots\} \). Similarly, if \( S = \{x/x^2 - 3x + 2 = 0\} \) then we insists that our set \( S \) should contains only the solutions to the given quadratic equation namely 1 and 2. i.e. \( S = \{1, 2\} \).

**Venn Diagram:** To make the analysis more comprehensive and easily understandable we often use compact pictorial representations of sets. Such pictorial representations are often called Venn diagrams.

**Example 6:** If \( A = \{a, b, c\} \) is the set under reference, then the corresponding pictorial Venn diagram is shown below. Here a small ellipse is used to represent the set. Within this ellipse all its members namely a, b and c are accommodated.

![Venn Diagram](image.png)

**Fig. 2.1**

**Universal set and sub-sets:** The whole of the given set is often called the universal set. The sub-set by definition is simply a portion or a part of the given universal set.
The Theory of Sets

Example 7: If \( U = \{1, 2, 3\} \) is the universal set then, \( A = \{1, 2\}; \ B = \{2, 3\}; \ C = \{3, 1\} \) are some of the sub-sets of \( U \) that one can frame. Further, since sub-sets by definition are partitions or parts of the universal set \( U \), it is quite but natural that they all must contained in \( U \), the universal set. To mean ‘contained in’ we use the notation ‘\( \subset \)’. Hence, in our illustration given above, since \( A, B \) and \( C \) are sub-sets of the universal set \( U \), we simply write \( A \subset U, B \subset U, C \subset U \) and so on.

Proper and improper sub-sets: A proper sub-set contains at least one element less than the given universal set. In this sense the proper sub-set is really a part and not the whole of the universal set \( U \). An improper sub-set however will have all the members of the universal set \( U \). In other words, the improper sub-set of the given universal set is the universal set itself.

Example 8: If \( U = \{a, b, c, d\} \) stands for the universal set, then \( A = \{a, b\}, \ B = \{a, b, c\} \) are some of the examples for the proper sub sets of \( U \).

If \( D = \{a, b, c, d\} \) then \( D \) is called the improper sub-set of \( U \) because \( D = U \).

Note: \( A \) is the proper sub-set of \( B \), then \( A \subset B \) and \( A \neq B \).

Singleton set: A set that contains only one element in it is often called a singleton set.

Example 9: \( A = \{1\}; \ B = \{b\}; \ C = \{Bangalore\} \) are some the examples of singleton sets.

Null or Empty set: A set, which contains no element of what so ever, is often called the null or empty set. It is normally denoted by the notation \( \phi \).

Example 10: If \( \phi = \{x/x \text{ is a human being in the moon}\} \) is called the empty set because there are no human beings in the moon as on today. Similarly, if \( \phi = \{x/x \text{ is a real number such that } x^2 + 1 = 0\} \), then again the solution set of the quadratic equation is empty because \( \pm \sqrt{-1} \) does not belongs to the real number set.

Set of sets: A set whose elements themselves are sets is called a set of sets.

Example 11: If \( A = \{\{3, 4\}, \{a, b\}, \phi\} \), then the set \( A \) is called the set of sets because in it the members themselves are sets.

Power set: The collection of all possible proper sub-sets of a given universal set \( U \) is called the power set of the universal set \( U \). Here the members themselves are sets of varying size. If the set \( A \) has ‘\( n \)’ elements in it then its power set will have altogether \( 2^n \) elements.

Example 12: If \( U = \{1, 2, 3\} \) then the set
\[
P = \{(1), (2), (3), (1, 2), (1, 3), (2, 3), (1, 2, 3), \phi\}
\]
is called the power set of the set \( A \). Here \( 2^3 = 8 \) elements are found in this power set.

2.3 Basic Operations on Sets

Like any other branch of Mathematics, in the theory of sets as well we have some important basic operations.

1. Differences among sets: If \( A \) and \( B \) are the two given sets then the difference set denoted by \( A – B \) is obtained by simply removing all the elements of the set \( B \) found in \( A \).
In all these diagrams (Fig. 2.2) the darkly shaded areas give us the needed set $A - B$.

By similar reasoning we define the set $B - A$ simply by removing all the elements of the set $A$ from the set $B$.

**Example 13:** If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$ then the set $A - B$ is defined as $A - B = \{1, 2\}$. Here the elements 3 and 4 found in B is removed from A.

**Example 14:** If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$ then the set $B - A$ is defined as $B - A = \{5, 6\}$.

**Note:** $A - B \neq B - A$, because the set $\{1, 2\} \neq \{5, 6\}$ in our illustration.

**Example 15:** If $A = \{a, b\}$ and $B = \{c, d\}$ then the set $A - B = A$. Here no element of B is found in A hence removal of elements does not arise.

**Difference of sets by using notations**

In symbolic form $A - B = \{x/x \in A \text{ but } x \notin B\}$

$$B - A = \{x/x \in B \text{ but } x \notin A\}$$

2. **Intersection of sets:** If $A$ and $B$ are the two sets then the intersection between $A$ and $B$ denoted by $A \cap B$ is defined as a third set having only the common elements of both the sets $A$ and $B$.

In all these diagrams (Fig. 2.3) the shaded areas, which are common to both $A$ and $B$, give us the needed set $A \cap B$.

**Example 16:** If $A = \{1, 2, 3, 4\}$ and $B = \{2, 3, 4, 5, 6\}$, then the set $A \cap B = \{2, 3, 4\}$. 